INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, 2013-14
Statistics - IV, Midterm Examination, March 10, 2014
Marks are shown in square brackets. Maximum Marks: 50
Answer as many questions as you wish.

1. Suppose $Z_1, \ldots, Z_5$ are i.i.d. $N(0, \sigma^2)$. Let $\mathbf{X} = (X_1, \ldots, X_5)'$ where $X_1 = Z_1$ and $X_{i+1} = X_i + Z_{i+1}$ for $1 \le i \le 4$.
(a) Find the probability distribution of $\mathbf{X}$.
(b) Find the probability distribution of $(X_5 - X_3)^2 + (X_3 - X_1)^2$. [10]

2. Consider an $I \times J$ contingency table where the (i, j) cell has probability $p_{ij} > 0$ for $1 \le i \le I$ and $1 \le j \le J$. Show that the row and column factors are independent if and only if

$\frac{p_{11}p_{ij}}{p_{1j}p_{i1}} = 1$ for all $i \neq 1$ and $j \neq 1$. [10]

3. Suppose X_1 and X_2 are i.i.d. $N(\mu, \sigma^2)$, and U be any nonnegative continuous random variable independent of X_1. Let $Y = X_1 + U$. Show that Y is stochastically larger than X_2. [10]

4. Suppose $D_1, D_2, \ldots, D_n$ are continuous random variables which are independent and are symmetric about 0. Let I_j be the indicator variable which is defined as $I_j = 1$ if $D_j \ge 0$ and 0 otherwise, for $1 \le j \le n$. Show that $(|D_1|, |D_2|, \ldots, |D_n|)$ and $(I_1, I_2, \ldots, I_n)$ are independently distributed. [10]

5. Let $X_1, X_2, \ldots, X_n$ be i.i.d. from a continuous distribution with c.d.f. F.
(a) Define the empirical distribution function F_n.
(b) Show that $F_n(x)$ converges in probability to $F(x)$ for each x as $n \to \infty$.
(c) Show that $\sup_{-\infty < x < \infty} |F_n(x) - F(x)|$ converges in probability to 0 as $n \to \infty$. [10]

6. Let $X_{(i)}^{(n)}$ denote the ith order statistic from a random sample of size n from a continuous distribution whose c.d.f. F is strictly increasing on the support of the distribution. Show that, for each fixed i,
$F(X_{(i)}^{(n)}) - \frac{i}{n} \longrightarrow 0$ in probability as $n \longrightarrow \infty$. [10]